**Formulating an Adaptive Scalaron Field Theory with Curvature Coupling**

**Introduction and Background**

Relativistic Field Theory (RFT) cosmology posits an **adaptive scalaron** – a scalar field $\phi$ whose properties vary with the local gravitational environment​file-4bzwyu5xwcza2f2huwkyos. In low-density cosmic voids, this single field behaves as a **“fuzzy” wave-like dark matter** (an ultralight boson, $m\sim10^{-22}$ eV) forming large coherent quantum condensates​file-4bzwyu5xwcza2f2huwkyos. In high-density regions (galaxy cores, clusters), the same field acquires a large effective mass and behaves like classical gravitating matter, or even modifies gravity in strong fields​file-4bzwyu5xwcza2f2huwkyos. In essence, dark matter and modified gravity emerge as **different phases of one field**, whose **effective mass and dynamics adapt to local density**​file-4bzwyu5xwcza2f2huwkyos. This idea synthesizes prior developments: ultralight axion-like dark matter forming kiloparsec-scale Bose–Einstein condensates​file-4bzwyu5xwcza2f2huwkyos, and scalar–tensor gravity theories (e.g. $f(R)$ gravity or chameleon fields) that use a scalar field coupled to curvature/matter to reproduce galactic dynamics​file-4bzwyu5xwcza2f2huwkyos. The RFT 9.x simulations have demonstrated this adaptive behavior: the scalaron remains coherent and wavelike in diffuse environments, decoheres into granular clumps in virialized halos, and can even undergo **collapse instabilities** when a critical mass density is exceeded. The goal now is to **formulate a field theory action** that captures this environmental adaptability in a clear, tractable way, paving the path for a formal RFT 10.0 theory paper.

To proceed, we review relevant scalar–tensor frameworks and then propose minimal **curvature-coupled action terms** that endow the scalaron with an environment-dependent effective mass. We will evaluate which formulations best reproduce the **coherence–decoherence–collapse thresholds** observed in simulations, construct a **phase-space diagram** delineating the scalaron’s regimes, and identify an appropriate **order parameter for quantum coherence**. Throughout, we draw analogies to condensed matter (Bose–Einstein condensates, phase transitions, entropy flow) to illuminate the physical picture. Finally, we discuss how certain couplings may naturally incorporate **entropy production and an arrow of time**, connecting the scalaron’s behavior with thermodynamic concepts.

**Candidate Coupling Structures for an Adaptive Scalaron**

A central challenge is to incorporate **environmental dependence** (local matter density or curvature) into the scalaron’s action in a minimal, physically motivated way. We seek coupling terms between $\phi$ and spacetime curvature (Ricci scalar $R$ or the matter stress–energy trace $T$) such that the **scalaron’s effective mass $m\_{\rm eff}(\phi)$ increases in dense regions** (suppressing quantum coherence) and decreases in diffuse regions (allowing a light, long-range field). Several candidate coupling schemes are considered:

* **Brans–Dicke–type Nonminimal Coupling ($\phi^2 R$)**: The scalaron directly multiplies the Ricci scalar in the action. A simple form is adding a term $\frac{1}{2},\xi,\phi^2 R$, where $\xi$ is a dimensionless coupling. In the Jordan-frame action this modulates the effective gravitational constant ($G\_{\rm eff}\sim1/\phi^2$), reminiscent of Brans–Dicke theory in which $1/G$ is promoted to a dynamic scalar field​file-4bzwyu5xwcza2f2huwkyos. *Physical effect:* In a region of high curvature ($R$ large, typically high matter density), the coupling $\xi\phi^2R$ drives the scalaron’s field equation to enforce a smaller $\phi$ (since the $R,\phi^2$ term contributes to the scalaron’s effective potential). This can be seen by treating $R$ as an external parameter: the term acts like a density-dependent mass term, $V\_{\rm eff}\supset \frac{1}{2}\xi R,\phi^2$. In equilibrium one finds $(m^2+\xi R)\phi \approx 0$, so a large $R$ yields $\phi\to0$ or an effectively heavy mass $\sqrt{m^2+\xi R}$ that suppresses $\phi$ fluctuations. **Thus, $\xi \phi^2 R$ coupling provides a built-in “chameleon” effect** wherein high curvature (dense) environments give the scalaron a large mass and small amplitude, while in low-curvature voids $\phi$ can be light and significant. This is analogous to Brans–Dicke varying-$G$ models, but here we emphasize using $\xi$ small enough to evade solar-system bounds (since any spatial variation in $G\_{\rm eff}$ is tightly constrained​file-4bzwyu5xwcza2f2huwkyos). We note that such nonminimal couplings can be rewritten in Einstein frame as a running mass or potential for $\phi$, making the connection to environment-dependent mass explicit.
* **$f(R)$ Gravity and the Scalaron as $f'(R)$**: $f(R)$ theories of gravity introduce a function of the Ricci scalar in the action, $S=\int d^4x,\sqrt{-g}, [\frac{1}{16\pi G}f(R)+L\_m]$. These can be recast as a scalar-tensor theory where the extra scalar degree of freedom $\Phi = f'(R)$ plays the role of our scalaron. The action becomes $S=\int \sqrt{-g}[ \frac{1}{16\pi G}( \Phi R - U(\Phi) ) + L\_m]$, with a potential $U(\Phi)$ determined by the form of $f(R)$. Notably, viable $f(R)$ models (e.g. Hu–Sawicki, Starobinsky models) are designed to exhibit the **chameleon mechanism**: the scalaron $\Phi$ acquires a heavy mass in high-density regions, thereby hiding its effects in the solar system, but is light in low-density cosmic environments​[arxiv.org](https://arxiv.org/abs/1711.08991#:~:text=,the%20chameleon%20mechanism%2C%20we%20calculate). In other words, **the $f(R)$ scalaron’s effective mass increases with ambient matter density**, just like the adaptive behavior we need. Indeed, $f(R)$ gravity can be “tuned” so that in galaxies the scalaron mediates only a short-range force (mimicking dark matter’s localized gravity), while in intergalactic space it mediates a long-range modification (mimicking a mild MOND-like effect)​[arxiv.org](https://arxiv.org/abs/1711.08991#:~:text=,the%20chameleon%20mechanism%2C%20we%20calculate). This makes $f(R)$ an attractive theoretical springboard: the *scalaron* in RFT might essentially be an $f(R)$-type scalar field that is massive in halos and light in voids. Mathematically, one could allow $f$ to depend also on $\phi$ explicitly, i.e. an $f(R,\phi)$ with mixed partials chosen to enforce the desired mass variation. However, the simplest approach is to use the well-studied equivalence of $f(R)$ to a scalar field with a density-dependent effective potential. We will leverage results from $f(R)$ models (e.g. thin-shell conditions, stability criteria) to guide our coupling design.
* **Chameleon Coupling to the Matter Trace ($f(T)$ or $\phi T$)**: The **chameleon mechanism** provides a direct template for environment-dependent mass. Here, the scalaron interacts with matter such that the effective potential is $V\_{\rm eff}(\phi) = V(\phi) + \frac{1}{M}\phi,T$ (in Einstein frame), where $T=\rho-3p\approx\rho$ is the trace of the stress tensor for nonrelativistic matter and $M$ is a coupling scale. The result is that **the scalar’s mass becomes a function of local matter density**: in high-density regions the extra linear term dominates and forces $\phi$ to a small equilibrium value, with large curvature $V''(\phi)$ (i.e. large $m\_{\rm eff}$), whereas in low-density regions $\phi$ settles at a larger value with a shallow curvature (small $m\_{\rm eff}$)​[en.wikipedia.org](https://en.wikipedia.org/wiki/Chameleon_particle#:~:text=weakly%20than%20gravity%2C,chameleon%20is%20able%20to%20evade)​[en.wikipedia.org](https://en.wikipedia.org/wiki/Chameleon_particle#:~:text=In%20most%20theories%2C%20chameleons%20have,displaystyle%20%5Calpha%20%5Csimeq%201). In fact, *“chameleon particles”* were defined precisely as scalar fields whose mass **increases with ambient energy density**​[en.wikipedia.org](https://en.wikipedia.org/wiki/Chameleon_particle#:~:text=weakly%20than%20gravity%2C,chameleon%20is%20able%20to%20evade). This mechanism aligns perfectly with the adaptive scalaron concept. Implementing it in the action can be done by a **nonminimal matter coupling**: e.g. coupling $\phi$ to the matter Lagrangian or including an $f(R,T)$ term. An explicit example is to include an interaction term $-\frac{1}{2}\alpha,\phi^2 T$ or $\alpha,\phi,T$ in the effective Lagrangian. Expanding for small perturbations, one finds $m\_{\rm eff}^2(\phi)\approx m\_0^2 + \alpha,T$​file-4bzwyu5xwcza2f2huwkyos (for linear coupling), so that a large local $T$ (high $\rho$) boosts the effective mass◆. **By design, this makes the scalaron “know about its environment”**​file-4bzwyu5xwcza2f2huwkyos. The chameleon approach has been widely studied in dark energy and modified gravity contexts, so we can borrow stability conditions (e.g. the field must have a thin-shell in massive bodies​[arxiv.org](https://arxiv.org/abs/1711.08991#:~:text=,the%20chameleon%20mechanism%2C%20we%20calculate)) to ensure a realistic $\alpha$ value. A variant is the **Symmetron mechanism**, wherein the scalar has a density-dependent symmetry-breaking potential: at high density, $\phi=0$ is the stable vacuum (suppressing any fifth force), while below a critical density, $\phi$ spontaneously acquires a nonzero vacuum expectation value (restoring a long-range force)​[link.springer.com](https://link.springer.com/article/10.1007/s10714-023-03070-2#:~:text=Penrose%20argues%C2%A0,a%20low%20value%20value%20of). The symmetron essentially gives the scalar a negative mass-squared in low-density environments (triggering a phase transition). This yields an abrupt (second-order) shift in $\phi$ behavior at a threshold density, which could map to a sharper coherence–decoherence transition. We mention this for completeness: the RFT scalaron could in principle be realized as a symmetron-type field tuned so that galactic densities lie near the symmetry-breaking threshold. However, to keep our model mathematically minimal, we prioritize the smoother chameleon-style coupling unless a sharp phase transition is needed to match simulations.
* **$f(R,T)$ and Generalized Geometry–Matter Couplings**: In 2011, Harko et al. proposed an $f(R,T)$ gravity where the Lagrangian is a function of both the Ricci scalar and the trace of the energy–momentum tensor​[arxiv.org](https://arxiv.org/abs/1104.2669#:~:text=,We). Such a coupling implies that “standard” matter is not separately conserved – energy can transfer between the scalar curvature sector and matter. While this approach is less common, it provides another way to achieve environmental sensitivity. For example, a term linear in $T$ effectively reproduces the chameleon coupling above; more complex $f(R,T)$ forms could introduce additional dependencies (e.g. making the coupling vary with cosmic epoch or environment). One must be cautious: $f(R,T)$ models generally predict an extra force on matter (non-geodesic motion) and can violate equivalence principle tests unless the function is carefully chosen​[arxiv.org](https://arxiv.org/abs/1104.2669#:~:text=and%20of%20the%20trace%20of,energy)​[arxiv.org](https://arxiv.org/abs/1104.2669#:~:text=particles%20are%20also%20obtained%20from,perihelion%20precession%20of%20the%20planet). Nonetheless, we consider $f(R,T)$ a broad framework that includes effective fluid interactions. It is intriguing from a thermodynamic perspective: the non-conservation of stress-energy in $f(R,T)$ can be viewed as an irreversible process (e.g. matter entropy production), perhaps offering a handle on emergent **time asymmetry** (discussed later). For our purposes, a simpler representative is to include a direct $\phi$–matter coupling in Einstein frame (as above) rather than fully general $f(R,T)$.

**Evaluation of Coupling Candidates:** All the above mechanisms share the qualitative trait needed: *the scalaron’s mass or vacuum expectation is tied to the local density/curvature*. The **Brans–Dicke $\phi^2R$ coupling** achieves this indirectly via the gravitational sector (making $\phi$ part of the geometry), while the **chameleon $\phi T$ coupling** does so via the matter sector. In practice, these can be made mathematically equivalent by field redefinitions (Jordan vs. Einstein frame). $f(R)$ models are known to act as chameleons​[arxiv.org](https://arxiv.org/abs/1711.08991#:~:text=,the%20chameleon%20mechanism%2C%20we%20calculate), so it’s unsurprising that all paths converge to similar behaviors. The simplest analytic handle might be the **explicit $m\_{\rm eff}^2(\phi) = m\_0^2 + \alpha,T$ form**​file-4bzwyu5xwcza2f2huwkyos, which we can integrate into an action by including a term like $\frac{\alpha}{2}\phi^2 T$ (in Jordan frame) or a potential term $-\alpha \rho \phi$ (in Einstein frame effective potential). We will use this as a toy model for deriving threshold conditions. Meanwhile, the **$\xi \phi^2 R$ coupling** will be useful for examining effects on cosmological curvature (e.g. does $\phi$ significantly alter the Friedmann equations in voids? likely small if $\xi$ is small​file-4bzwyu5xwcza2f2huwkyos).

**Figure below** illustrates one coupling scenario (chameleon-like) in terms of the scalaron’s effective potential $V\_{\rm eff}(\phi)$ in different environments. In low-density settings, the effective potential (including self-interaction $V(\phi)$ and coupling to background matter density $\rho$) has a minimum at a relatively large $\phi$ with a flat curvature (light $m\_{\rm eff}$). In high-density settings, the additional coupling term tilts the potential, shifting the minimum to $\phi\approx 0$ with a steep curvature (heavy $m\_{\rm eff}$). This demonstrates how a single scalar field can **self-adjust from a light field value in voids to a heavy, suppressed value in dense regions**.

*Effective potential $V\_{eff}(\phi)$ for a chameleon-like scalaron in low vs. high matter density environments (schematic). The red curve (high density, large $T$ or $R$) has a minimum at a small field value $\phi\_{\rm eq,\ high}$, with a steep curvature (implying a large effective mass $m\_{\rm eff}$ and thus short-range fluctuations). The green curve (low density) has its minimum at a larger $\phi\_{\rm eq,\ low}$ with a shallow curvature (small $m\_{\rm eff}$, long-range field). Such curvature-coupled interactions give the scalaron a* ***self-adjusting mass*** *that increases with ambient density​*[*en.wikipedia.org*](https://en.wikipedia.org/wiki/Chameleon_particle#:~:text=weakly%20than%20gravity%2C,chameleon%20is%20able%20to%20evade)*. In high-density regions (galaxy cores, etc.), the field effectively “hides” (acquiring a large mass and negligible amplitude)​*[*en.wikipedia.org*](https://en.wikipedia.org/wiki/Chameleon_particle#:~:text=weakly%20than%20gravity%2C,chameleon%20is%20able%20to%20evade)*, whereas in low-density cosmic voids it remains light and can attain a significant value. This mechanism underlies the* ***chameleon effect****​*[*en.wikipedia.org*](https://en.wikipedia.org/wiki/Chameleon_particle#:~:text=weakly%20than%20gravity%2C,chameleon%20is%20able%20to%20evade) *and is one way to implement the adaptive scalaron behavior.*

**Coherence, Decoherence, and Collapse: Matching Simulation Thresholds**

Armed with candidate coupling frameworks, we next assess how well they reproduce the **qualitative thresholds observed in RFT 9.x simulations** – namely the transitions between a coherent wave-dominated scalaron, a decoherent classical-like state, and a gravitational collapse instability. In the simulations (both Schrödinger–Poisson and full GR runs), these transitions manifest as the halo environment changes​file-3zh15rq3mb1bnnjszwe2yx​file-3zh15rq3mb1bnnjszwe2yx:

* **Coherent regime:** In infant or low-mass halos (or the cosmic web filaments/voids), the scalaron remains as a **large-scale coherent wavefunction** – essentially a Bose–Einstein condensate (BEC) enveloping the system. All or most of the scalaron’s mass is in a single quantum state with a well-defined global phase. For example, early in halo collapse the entire dark matter halo can oscillate in unison as a “quantum pressure”-supported condensate​file-3zh15rq3mb1bnnjszwe2yx. This corresponds to a **coherence fraction** (fraction of mass in the condensate mode) near unity. The simulations track this via phase correlations and the off-diagonal components of the one-particle density matrix​file-3zh15rq3mb1bnnjszwe2yx.
* **Decoherent (classical) regime:** As the halo grows and virializes, gravitational interactions and wave interference cause the scalaron to **dephase and fragment** into many excited modes (sometimes described as a “granular” or wave-turbulent state)​file-3zh15rq3mb1bnnjszwe2yx​file-3zh15rq3mb1bnnjszwe2yx. The field’s single-phase description breaks down except perhaps in the very center. This is essentially the field transitioning to behave like classical collisionless dark matter – numerous independent lumps with randomized phases, akin to particles. The **coherence fraction drops** (e.g. only the central solitonic core might remain phase-coherent, containing a small fraction of the total mass, while the outer halo is incoherent)​file-3zh15rq3mb1bnnjszwe2yx. In simulation, this corresponds to the point where interference fringes wash out and the one-body density matrix indicates a mixed state. Notably, this **decoherence is driven internally by gravity** – no external “observer” is measuring the wavefunction, but the many-body gravitational chaos serves as an effective environment​file-4bzwyu5xwcza2f2huwkyos. As noted in RFT 9.0, “gravity itself provides the decohering bath” for the scalaron​file-4bzwyu5xwcza2f2huwkyos, analogous to how a heat bath causes a quantum system to lose phase coherence.
* **Collapse (unstable) regime:** In extreme cases, the scalaron can undergo a **gravitational collapse** akin to a boson star instability (sometimes called a “bosenova” by analogy to collapsing BECs). Simulations provoke this by pushing cores beyond the maximum mass supportable by quantum pressure​file-3zh15rq3mb1bnnjszwe2yx. When a scalaron core’s self-gravity overwhelms both quantum pressure and any dispersion, it undergoes runaway collapse, potentially forming a black hole (or a dense oscillatory remnant)​file-3zh15rq3mb1bnnjszwe2yx​file-3zh15rq3mb1bnnjszwe2yx. The hallmark is a sudden spike in central density and a burst of scalar field radiation as the core “boils off” excess mass​file-3zh15rq3mb1bnnjszwe2yx. Two pathways to collapse were identified: (i) a **quasi-static mass build-up** (e.g. accretion or merger) causes the core to exceed the critical mass; (ii) a **dynamic perturbation** (like a tidal shock or rapid compression) pushes a metastable core over the edge​file-3zh15rq3mb1bnnjszwe2yx. In either case, there appears to be a **critical threshold** in parameter space demarcating stable vs. collapse outcomes​file-3zh15rq3mb1bnnjszwe2yx. Analytically, for an isolated fuzzy dark matter core with no self-interaction, the threshold is on the order of the Chandrasekhar limit for a boson star: $M\_{\rm crit}\sim0.6,M\_{\rm Pl}^2/m$​file-3zh15rq3mb1bnnjszwe2yx (about $10^{12} M\_\odot$ for $m=10^{-22}$ eV​file-3zh15rq3mb1bnnjszwe2yx). In the adaptive scalaron model, additional interactions (from curvature coupling) or departures from isolation can reduce this critical mass, allowing collapse at smaller scales​file-3zh15rq3mb1bnnjszwe2yx​file-3zh15rq3mb1bnnjszwe2yx. One goal of the theory is to encode such **instability criteria** in the field equations.

**Reproducing these regimes with coupling models:** The coupling structures proposed must be able to generate (or at least allow) the above transitions as the environment changes. Broadly, the **coherent–decoherent transition** should occur when the scalaron’s **de Broglie wavelength $\lambda\_{\rm dB}$ becomes comparable to (or smaller than) the system’s characteristic scale** (interfering many modes within the halo), or equivalently when the coherence length falls below the region size. In a gravitational halo, $\lambda\_{\rm dB} \sim \frac{h}{m v}$ is set by the particle velocity dispersion $v$; higher-density halos have deeper potentials and higher $v$, thus smaller $\lambda\_{\rm dB}$. This naturally explains why low-density environments (voids, dwarf galaxies) retain wave coherence – their velocity dispersions are tiny, making $\lambda\_{\rm dB}$ large (kiloparsecs), larger than the system, so the field can stay in a coherent ground state. In massive halos or cluster cores, $v$ is large (hundreds or thousands of km/s), giving $\lambda\_{\rm dB}$ of order parsecs or less, which is much smaller than the halo – hence many uncorrelated wave packets fit inside, yielding an effectively classical ensemble.

The **couplings to curvature/matter enhance this effect** by providing an *additional source of decoherence*: as density increases, $m\_{\rm eff}$ of the scalaron increases (from the chameleon mechanism), which **shrinks the coherent length** even more. For instance, if $m\_{\rm eff}$ becomes large in a dense halo, the field’s Compton wavelength $\sim m\_{\rm eff}^{-1}$ becomes tiny, cutting off large-scale quantum coherence. Thus, the environment-triggered mass makes the quantum-to-classical transition more pronounced. In practical terms, one could define a **critical density $\rho\_{\rm deco}$** (or critical curvature $R\_{\rm deco}$) at which $m\_{\rm eff}(\rho)$ times the halo size exceeds some value (meaning many oscillation modes fit), causing decoherence. The **$\phi T$ coupling model naturally yields such a condition**: solve $m\_0^2 + \alpha \rho\_{\rm deco} = k^2$ (where $k$ relates to inverse system size or dynamical time), to find $\rho\_{\rm deco}$. If $\alpha$ is tuned such that this density corresponds to, say, the typical halo virial density, then all halos above that will mostly decohere. The **$\xi \phi^2R$ model** similarly contributes – high $R$ in a deep potential well effectively adds to $m\_{\rm eff}^2$. In summary, the couplings help **set a quantitative threshold for coherence loss** that can match simulation observations (e.g. halo mass or velocity dispersion at which granulation appears).

For the **collapse threshold**, the couplings play a more nuanced role. On one hand, a very large $m\_{\rm eff}$ in dense regions tends to *stabilize* the field against forming a single large condensate – the field fragments instead of concentrating, which might avert collapse in many situations. On the other hand, if a region manages to remain in a coherent state despite high density (e.g. a central soliton core which is stabilized by self-gravity but still quantum-coherent), the added self-interaction from curvature coupling could act as an **effective attraction** that lowers the collapse mass. For example, if the scalaron has a coupling $-\alpha \phi^2 R$, as $\phi$ grows in a massive core, it slightly reduces the local Ricci scalar (since $\phi$ carries stress-energy), possibly weakening the effective pressure support. Alternatively, considering an *Einstein-frame* picture: a term like $\alpha \phi^2 R$ can be reinterpreted as an extra potential $U(\phi)\propto \alpha \phi^4$ in the scalar field equation (after scaling out the gravitational part). A positive $\alpha$ in $\xi \phi^2R$ typically acts like a **repulsive (stabilizing) quartic** interaction (hence why the Higgs inflation model uses $\xi\phi^2R$ to flatten the potential). However, a negative $\xi$ would act like an **attractive self-interaction**, hastening collapse. Thus, by choosing the sign of coupling constants, one can in principle adjust whether the adaptive effects delay collapse or facilitate it. The RFT simulations suggest that **collapse can occur** under realistic conditions (e.g. cluster-scale solitons, or axion self-interactions)​file-3zh15rq3mb1bnnjszwe2yx​file-3zh15rq3mb1bnnjszwe2yx, so our model should accommodate that. Likely, a small effective attractive self-interaction is present – either from the scalar’s own potential $V(\phi)$ or induced via coupling (e.g. if a symmetron symmetry-breaking occurs, it can introduce a $\phi^4$ term with negative coefficient near $\phi=0$). We will examine the stability of solutions by linear perturbation analysis to delineate the collapse boundary in each coupling scenario.

In conclusion, by appropriate choice of coupling parameters $(\xi,\alpha,$ etc.), the **adaptive scalaron action can reproduce a sequence**: at $\rho \ll \rho\_{\rm deco}$ (or shallow $R$), $\phi$ is light and forms a coherent condensate; at $\rho \sim \rho\_{\rm deco}$, $m\_{\rm eff}$ grows to $\sim$halo dynamical scale, coherence breaks down; and if a localized region pushes to $\rho > \rho\_{\rm crit}$ while still coherent (large $\phi$), collapse ensues. These regimes can be summarized in a **phase diagram** as a function of environmental parameters, which we construct next.

**Phase-Space Diagram of Scalaron Regimes**

We now map out the scalaron’s phases (coherent, decoherent, collapse) in a **phase-space diagram** using two key variables: **local matter density** (or an equivalent like gravitational potential depth) and **quantum coherence scale** (e.g. de Broglie wavelength or coherence length). This visualization will clarify the boundaries between regimes. **Figure below** presents a qualitative version of such a diagram, informed by both physical estimates and simulation insights:

*Qualitative phase-space diagram for the scalaron, showing regions of different behavior as a function of* ***local density*** *(x-axis, increasing rightward) and* ***de Broglie wavelength*** *$\lambda\_{\rm dB}$ (y-axis, decreasing downward). The dashed line denotes the* ***coherence threshold****: above this line (blue shaded region* ***A****), $\lambda\_{\rm dB}$ is large relative to the region (or equivalently, the scalaron’s coherence length is long), so the field stays in a* ***Coherent BEC (wave-like) phase****. Below the line (region* ***B****, shaded light purple), $\lambda\_{\rm dB}$ is too small to maintain a single phase across the region, leading to a* ***Decoherent, classical-like phase****. At extremely high densities (far right), an additional region* ***C*** *(red) indicates the* ***Collapse/unstable regime****, where self-gravity overwhelms quantum pressure. The collapse region appears at the upper end of densities and for configurations that remained sufficiently coherent (i.e. it lies near the boundary of A and B): a coherent dense core can cross into collapse, whereas a fully decoherent configuration at the same density would virialize rather than collapse. This diagram can be used to predict, given a halo’s density (or velocity dispersion) and the scalaron’s $\lambda\_{\rm dB}$, which regime the scalaron will be in​file-4bzwyu5xwcza2f2huwkyos​file-3zh15rq3mb1bnnjszwe2yx. For instance, a dwarf galaxy (low density, moderate $\lambda\_{\rm dB}$) lies in A (coherent core), a Milky-Way halo (higher density, smaller $\lambda\_{\rm dB}$) lies in B (decoherent granular halo), and an extreme cluster core might approach C (collapse-prone) if a large condensate tries to form.*

In this conceptual diagram, the **coherence threshold line** (dashed) can be interpreted via an equality like $\lambda\_{\rm dB} \sim L\_{\rm system}$ or by a critical coherence fraction. Above the line, the scalaron exhibits **macroscopic quantum coherence** – meaning a large fraction of particles occupy the ground state wavefunction, and quantum interference is manifest on macroscopic scales​[en.wikipedia.org](https://en.wikipedia.org/wiki/Bose%E2%80%93Einstein_condensate#:~:text=fraction%20Image%3A%20,At%20temperatures%20near). Below the line, the system is effectively a classical mixture (many excited states populated, random phases). The exact slope of this threshold line comes from our coupling model: if $m\_{\rm eff}$ increases rapidly with density, the line becomes steeper (pushing coherence to lower densities). In the drawn version, we assume a moderate scaling roughly $\lambda\_{\rm dB}\propto \rho^{-1/2}$ (as would hold if $\lambda\_{\rm dB}\sim 1/m\_{\rm eff} v$ and $v^2\sim \rho^{2/3}$ for virialized halos). The **collapse region (C)** is shown as a lobe at high density overlapping the coherent side. This reflects that to undergo collapse, the scalaron must **remain (at least partly) coherent up to the point of instability**​file-3zh15rq3mb1bnnjszwe2yx​file-3zh15rq3mb1bnnjszwe2yx. In practice, a halo entering B (decoherent) will typically **avoid collapse** by virializing into a stable clumpy halo. But if a condensate core persists (A) and grows too dense, it can hit the collapse boundary and transition to C. The red region’s placement is guided by the critical core mass condition from simulations​file-3zh15rq3mb1bnnjszwe2yx​file-3zh15rq3mb1bnnjszwe2yx. We see that extremely massive halos (far right) might reach it – consistent with the finding that an isolated $10^{12}M\_\odot$ fuzzy DM core would collapse​file-3zh15rq3mb1bnnjszwe2yx, though ordinary structure formation might not produce such a single coherent core. However, with the scalaron’s adaptive interactions, collapse could occur at somewhat lower masses (e.g. if attractive self-coupling effectively lowers $M\_{\rm crit}$​file-3zh15rq3mb1bnnjszwe2yx).

One can also annotate the **coherence fraction** $f\_{\rm coh}$ or an **order parameter** on such a diagram. In region A, $f\_{\rm coh}\approx 1$ (almost all particles in the condensate mode), whereas in B, $f\_{\rm coh}\ll 1$. Region C might be thought of as an extrapolation of A beyond stability (so $f\_{\rm coh}\approx 1$ until collapse happens, after which the concept breaks down or $f\_{\rm coh}$ resets to 0 if a black hole forms). In a sense, $f\_{\rm coh}$ plays a role analogous to a **thermodynamic order parameter** distinguishing a “quantum condensed phase” from a “classical normal phase.” We explore this analogy next.

**Coherence Order Parameter and Phase Entropy**

A useful concept from condensed matter is the **order parameter** that characterizes phases and their transitions. For a Bose–Einstein condensate, the condensate wavefunction $\psi$ (or the condensate fraction $n\_0/N$) serves this role: in the condensed phase, $\psi \neq 0$ (macroscopic occupancy of ground state), while in the normal phase, $\psi = 0$ (no single dominant phase-coherent state)​[en.wikipedia.org](https://en.wikipedia.org/wiki/Bose%E2%80%93Einstein_condensate#:~:text=fraction%20Image%3A%20,At%20temperatures%20near)​[en.wikipedia.org](https://en.wikipedia.org/wiki/Bose%E2%80%93Einstein_condensate#:~:text=negligible%20fraction%20of%20the%20total,how%20small%20the%20energy%20difference). By analogy, we define a **coherence fraction** $F\_c$ for the scalaron, e.g. Fc  ≡  McohMtotal,F\_c \;\equiv\; \frac{M\_{\rm coh}}{M\_{\rm total}},Fc​≡Mtotal​Mcoh​​, where $M\_{\rm coh}$ is the mass in the largest coherent mode (e.g. the ground-state or solitonic component) and $M\_{\rm total}$ is the total mass of scalaron in the region of interest. In region A (coherent phase), $F\_c \approx 1$ or is at least significantly nonzero. In region B (decoherent phase), $F\_c$ drops toward 0 – the largest coherent structure (perhaps a small core) contains only a tiny fraction of the mass. Thus, $F\_c$ can serve as an order parameter, being high in one phase and near-zero in the other​file-4bzwyu5xwcza2f2huwkyos. A continuous decrease of $F\_c$ with increasing density or temperature would indicate a second-order (continuous) quantum-to-classical transition, whereas a sudden drop at a threshold (perhaps in a symmetron-like scenario) would indicate a more first-order transition. Simulations indeed measure something akin to $F\_c$ (by diagonalizing the one-body density matrix and looking at the largest eigenvalue)​file-4bzwyu5xwcza2f2huwkyos.

Alongside $F\_c$, we can define a **phase entropy** $S\_{\rm phase}$ to quantify the “disorder” in the scalaron’s phase distribution. In a pure coherent state, $S\_{\rm phase}$ is minimal (in an ideal pure BEC at zero temperature, the entropy is zero​[nii.ac.jp](https://www.nii.ac.jp/qis/first-quantum/e/forStudents/lecture/pdf/qis385/QIS385_chap2.pdf#:~:text=Below%20the%20BEC%20phase%20transition,which%20an%20entropy%20is%20zero)). As phases randomize, entropy increases. One way to estimate this is via the Shannon entropy of the phase angle distribution or the von Neumann entropy of the one-particle density matrix. In RFT 9.0, it was noted that quantum decoherence is effectively an **entropy-increasing process** – the initially pure (low entropy) state evolves into a mixed (higher entropy) state​file-4bzwyu5xwcza2f2huwkyos. We can formalize: if $\hat{\rho}$ is the density matrix of the scalaron’s coarse-grained state, then the **von Neumann entropy** $S = -\mathrm{Tr}(\hat{\rho}\ln\hat{\rho})$ serves as a measure of decoherence. In a pure condensate, $\hat{\rho}=|\Psi\rangle\langle\Psi|$ has $S=0$. In a statistical mixture of many modes, $S>0$. The growth of $S$ as structures form is essentially the *emergence of the arrow of time* in this system​file-4bzwyu5xwcza2f2huwkyos​file-4bzwyu5xwcza2f2huwkyos. We may even identify **$S\_{\rm phase}$ as a macroscopic time parameter**: as the universe (or a halo) evolves, $S\_{\rm phase}$ increases, indicating the direction of time (from coherent past to decoherent future). The coherence fraction $F\_c$ and entropy $S\_{\rm phase}$ are directly related: $F\_c=1$ corresponds to $S\_{\rm phase}$ minimal; as $F\_c\to 0$, $S\_{\rm phase}$ approaches maximum (for a given energy). This mimics the role of the condensate fraction vs. entropy in finite-temperature BECs (below $T\_c$, a fraction condenses and entropy is lower than a classical gas at the same energy​[nii.ac.jp](https://www.nii.ac.jp/qis/first-quantum/e/forStudents/lecture/pdf/qis385/QIS385_chap2.pdf#:~:text=Below%20the%20BEC%20phase%20transition,which%20an%20entropy%20is%20zero)).

From a **field-theoretic perspective**, incorporating $F\_c$ or $S\_{\rm phase}$ is challenging because standard actions do not have explicit entropy terms. However, one approach is an **effective field theory for the condensate mode coupled to a “gas” of excitations**. One could write $\phi = \phi\_0(t) + \varphi$ where $\phi\_0$ is the coherent mean field (order parameter) and $\varphi$ represents fluctuations/excited modes. Tracing out $\varphi$ (which act like an environment for $\phi\_0$) would yield an effective equation for $\phi\_0$ that is *dissipative* and breaks time-reversal symmetry – capturing decoherence and entropy production. In essence, the coupling of $\phi$ to curvature/matter provides channels for $\phi\_0$ to lose coherence into either matter degrees or gravitational radiation. While we won’t explicitly derive an open-system master equation here, this conceptual picture is consistent with the idea that “gravity acts as a bath” causing $\phi\_0$’s off-diagonal density matrix elements to decay​file-4bzwyu5xwcza2f2huwkyos. The **phase entropy $S\_{\rm phase}$ thus increases with time**, and one could postulate an $H$-theorem for it: $dS\_{\rm phase}/dt \ge 0$ for an isolated gravitating scalaron (barring Poincaré recurrences in an ideal mathematical sense). This would align with what we see in simulations: as structure forms and chaos develops, the system’s state becomes more mixed and information about the initial phase alignment is effectively lost​file-4bzwyu5xwcza2f2huwkyos​file-4bzwyu5xwcza2f2huwkyos.

In summary, we treat **$F\_c$ (coherence fraction) as the order parameter** distinguishing the scalaron’s quantum vs. classical phases. When $F\_c$ falls below some threshold (say 0.5), we declare the field decohered. This has “field-theoretic consequences” in that the effective stress-energy of the field changes: a fully coherent scalar field has stress-energy resembling a superfluid (low entropy, possibly anisotropic stress from the condensate’s quantum pressure), whereas a decoherent collection of waves behaves like a classic ensemble (with an isotropic pressure dispersion and higher entropy). Indeed, as $F\_c$ drops, the **equation of state** of the scalaron matter should transition from that of a condensate (nearly cold, with quantum pressure behaving like an extra term) to that of an effectively warm collisionless gas. This transition could be encoded by making the stress tensor of $\phi$ depend on $F\_c$ or $S\_{\rm phase}$ (for instance, adding an **entropy term to the effective fluid pressure**). Some researchers have drawn analogies between virialized halo dark matter and a finite-temperature Bose gas, where the “granule” fluctuations correspond to an effective temperature. One could envision an **effective free energy** $\mathcal{F}(F\_c)$ for the scalaron: high $F\_c$ state being a local minimum at low “temperature” (low environmental perturbations), which disappears or becomes unstable at high perturbation (yielding $F\_c\to0$).

**Entropy and the Arrow of Time in the Scalaron Dynamics**

The emergence of **time’s arrow** in cosmology is tightly linked to entropy production. In our model, the scalaron’s behavior provides a microcosm of this: the **early universe or cosmic voids** start with the scalaron in a very low-entropy, ordered state (homogeneous coherent field, analogous to a zero-temperature BEC)​file-4bzwyu5xwcza2f2huwkyos​file-4bzwyu5xwcza2f2huwkyos. As time progresses, gravitational instability causes structure formation – perturbations grow, the field clusters and **decoheres, increasing the entropy** associated with the scalaron field​file-4bzwyu5xwcza2f2huwkyos​file-4bzwyu5xwcza2f2huwkyos. This mirrors the classic statement by Roger Penrose that the clumping of matter corresponds to an increase in gravitational entropy, providing the arrow of time in our universe​file-4bzwyu5xwcza2f2huwkyos. Initially, the universe (or a large region of it) had a very uniform scalaron field – a state of low gravitational entropy (all mass distribution is smooth)​file-4bzwyu5xwcza2f2huwkyos. Over time, that field clusters into galaxies and perhaps black holes, which are states of extremely high entropy (many possible microstates for the same macro distribution)​file-4bzwyu5xwcza2f2huwkyos. **Penrose’s argument** indeed states that a uniform distribution of matter has low entropy, whereas a lumpy one (especially one with black holes) has high entropy​[link.springer.com](https://link.springer.com/article/10.1007/s10714-023-03070-2#:~:text=Penrose%20argues%C2%A0,a%20low%20value%20value%20of). Our scalaron transitions from the former to the latter, thereby *defining a direction of time*: the direction in which structures grow and phases decohere is the future-directed time.

Crucially, none of the fundamental equations we wrote (the action principles with $\phi$ coupling to $R$ or $T$) explicitly break time-reversal symmetry – they are symmetric under time reversal at the microscopic level. The arrow arises because of initial conditions (the universe starts in an ordered state) and the **dynamical instability** that drives the system to explore a larger volume of phase space (i.e. more entropy). The **environment-induced decoherence** provides the microscopic irreversibility: once $\phi$ entangles with many gravitational degrees of freedom, the process cannot unspontaneously reverse​[en.wikipedia.org](https://en.wikipedia.org/wiki/Quantum_decoherence#:~:text=Decoherence%20%20can%20be%20viewed,with%E2%80%94or%20transferring%20it%20to%E2%80%94the%20surroundings)​[en.wikipedia.org](https://en.wikipedia.org/wiki/Quantum_decoherence#:~:text=loosely%20coupled%20with%20the%20energetic,with%E2%80%94or%20transferring%20it%20to%E2%80%94the%20surroundings). Technically, the combined system (scalaron + gravity) is closed and obeys unitary evolution, but from the perspective of $\phi$’s *subsystem*, information has leaked into gravitational fields (metric perturbations, high-frequency excitations) and is practically irretrievable​[en.wikipedia.org](https://en.wikipedia.org/wiki/Quantum_decoherence#:~:text=Decoherence%20%20can%20be%20viewed,with%E2%80%94or%20transferring%20it%20to%E2%80%94the%20surroundings)​[en.wikipedia.org](https://en.wikipedia.org/wiki/Quantum_decoherence#:~:text=plus%20environment%20evolves%20in%20a,with%E2%80%94or%20transferring%20it%20to%E2%80%94the%20surroundings). This is analogous to how friction converts ordered motion into heat – here gravitational mixing converts coherent wavefunction phases into “heat” in the form of complex phase space configurations. The result is **effectively non-unitary, irreversible dynamics for the scalaron alone**​[en.wikipedia.org](https://en.wikipedia.org/wiki/Quantum_decoherence#:~:text=Decoherence%20%20can%20be%20viewed,with%E2%80%94or%20transferring%20it%20to%E2%80%94the%20surroundings)​[en.wikipedia.org](https://en.wikipedia.org/wiki/Quantum_decoherence#:~:text=plus%20environment%20evolves%20in%20a,with%E2%80%94or%20transferring%20it%20to%E2%80%94the%20surroundings). We can say that the **entropy gradient $dS\_{\rm phase}/dt > 0$ defines the arrow of time** for the scalaron, just as rising entropy defines the thermodynamic arrow​[en.wikipedia.org](https://en.wikipedia.org/wiki/Entropy_as_an_arrow_of_time#:~:text=Entropy%20is%20one%20of%20the,system%20can%20increase%2C%20but%20not). In simpler terms: as structure forms, $F\_c$ decreases and $S\_{\rm phase}$ increases, telling us time is moving forward​file-4bzwyu5xwcza2f2huwkyos.

It is remarkable that a single scalar field model can encapsulate this cosmological arrow. Usually one discusses entropy increase in terms of cosmic gas, radiation, etc., but here even a “dark” scalar field (with no direct interactions except gravity) produces an entropy flow through gravitational interactions. This supports Penrose’s view that gravity itself has a huge capacity for entropy – initially, most of it was unactivated (homogeneous field, low gravitational entropy), but as gravity drives clustering, those gravitational degrees of freedom become excited (large inhomogeneities, possibly black holes), raising entropy​[link.springer.com](https://link.springer.com/article/10.1007/s10714-023-03070-2#:~:text=Penrose%20argues%C2%A0,a%20low%20value%20value%20of). The adaptive scalaron model makes this concrete: *gravity acts as a one-way drive that turns a pristine condensate into a high-entropy mixture*, aligning with the Second Law. In RFT 9.0 it was noted: “the growth of cosmic structures provides a concrete mechanism for decoherence: the many-body gravitational interaction plays the role of an environment that irreversibly scrambles the phases of the field. The process is unidirectional in time – once phases are randomized, they do not reassemble”​file-4bzwyu5xwcza2f2huwkyos​file-4bzwyu5xwcza2f2huwkyos. Our coupling choices do not change this fundamental irreversibility; however, certain formulations (like $f(R,T)$ gravity) might explicitly include non-conservative processes that resemble friction or particle production, making the arrow even more explicit. For instance, an $f(R,T)$ term can cause an **effective transfer of energy from $\phi$ to matter** (or vice versa), acting like a built-in dissipative process which ensures entropy increases (since such coupling is analogous to an open system).

In the end, whichever coupling scheme we adopt, it must be consistent with the universe’s thermodynamic arrow. The **entropy produced by scalaron decoherence should flow into either microscopic gravitational radiation or microscopic matter degrees** (e.g. dark matter heat, if any). This might offer a test: if the scalaron produces, say, nonthermal gravitational waves or excess particle excitations as it collapses or decoheres, those could carry away the “lost” phase information. Indeed, simulations of collapse show a burst of scalar radiation (wave energy) leaving the system​file-3zh15rq3mb1bnnjszwe2yx​file-3zh15rq3mb1bnnjszwe2yx – this is entropy leaving the condensate, making the irreversibility manifest. If we were to compute the entropy of those outgoing waves plus the remaining object, it should be higher than the initial condensate’s entropy, satisfying the Second Law. Thus, the **arrow of time emerges naturally**: forward in time = from coherent to incoherent, from low entropy to high entropy.

**Connections to Condensed Matter Analogies**

Throughout this discussion, we have highlighted analogies to **condensed matter systems** which help build intuition:

* **BEC Phase Transition:** The scalaron’s behavior is analogous to a Bose gas going through a condensation (or decondensation) transition. In the early universe or in voids, the scalaron is like a condensate well below the critical “temperature” (here, disorder due to gravity), so a **large fraction of bosons occupy the lowest state**​[en.wikipedia.org](https://en.wikipedia.org/wiki/Bose%E2%80%93Einstein_condensate#:~:text=large%20fraction%20of%20bosons%20occupy,macroscopic%20occupation%20of%20one%20or) and quantum interference is observable on large scales. In galaxy halos, as the system “warms up” (via virial motions and perturbations), it crosses a critical point where the condensate fraction drops and most particles are in excited states. This mirrors how a BEC loses condensate fraction as temperature rises above $T\_c$​[en.wikipedia.org](https://en.wikipedia.org/wiki/Bose%E2%80%93Einstein_condensate#:~:text=fraction%20Image%3A%20,At%20temperatures%20near)​[en.wikipedia.org](https://en.wikipedia.org/wiki/Bose%E2%80%93Einstein_condensate#:~:text=negligible%20fraction%20of%20the%20total,how%20small%20the%20energy%20difference). Our coherence fraction $F\_c$ is equivalent to the condensate fraction used in BEC theory, serving as an order parameter that goes from 1 to 0 across the transition. However, there are differences: the scalaron’s transition is driven not by temperature in the usual sense, but by self-gravitational interactions – a kind of **quantum-to-classical phase transition driven by gravity** rather than thermal contact. It is a dynamical, out-of-equilibrium transition, whereas laboratory BEC transitions are usually considered in equilibrium. Nonetheless, concepts like **critical slowing down** or phase-ordering dynamics might have analogues (e.g. as a halo virializes, how fast do phases decohere – possibly analogous to Kibble–Zurek mechanism for phase transition in an expanding universe? This could be a topic of further inquiry).
* **Superfluid and Normal Fluid Phases:** If one considers the scalaron in a halo as a fluid, then region A (coherent) is akin to a **superfluid phase** – it has long-range order (phase coherence) and can support non-classical phenomena (like the large-scale quantum pressure supporting solitonic cores). Region B (decoherent) is like a **normal fluid** or gas – essentially classical, no long-range order, subject to classical Jeans instability and virial motions. Indeed, one could attempt to apply the two-fluid model often used in superfluid helium: a fraction of the scalaron is a superfluid (condensate core) and the rest is a normal component (granular halo). The normal component carries entropy, the superfluid component does not. As the system evolves, the fraction of superfluid decreases (just as in helium as you raise temperature). Some works have even drawn analogies between fuzzy dark matter halos and superfluid dark matter, treating the granules as finite-temperature excitations. Our framework makes this analogy more concrete, by explicitly tying the fraction to environment density via couplings.
* **Decoherence and Entanglement:** In quantum computing or atomic experiments, decoherence is something to be avoided, but here it is the natural outcome of a complex self-interacting system. The scalaron’s wavefunction becomes **entangled with gravitational degrees of freedom** (in a sense, each region of the halo gets a different phase kicked by local gravitational potential fluctuations). This is similar to how a qubit entangles with its environment and loses coherence​[en.wikipedia.org](https://en.wikipedia.org/wiki/Quantum_decoherence#:~:text=Decoherence%20%20can%20be%20viewed,with%E2%80%94or%20transferring%20it%20to%E2%80%94the%20surroundings)​[en.wikipedia.org](https://en.wikipedia.org/wiki/Quantum_decoherence#:~:text=plus%20environment%20evolves%20in%20a,with%E2%80%94or%20transferring%20it%20to%E2%80%94the%20surroundings). The mathematics of environment-induced decoherence, developed by Zurek and others, could potentially be applied to quantify how fast the scalaron decoheres in a given environment. For instance, one could estimate a **decoherence timescale** $t\_{\rm deco}$ for a perturbation mode – perhaps on the order of a few dynamical times of the halo, since after a few crossing times phases get mixed. This is consistent with simulations that show halos largely losing coherence after virialization (a few crossing times)​file-3zh15rq3mb1bnnjszwe2yx​file-3zh15rq3mb1bnnjszwe2yx.
* **Bosenova and Critical Collapse:** The term *bosenova* is borrowed from laboratory BEC experiments where an attractive interaction (negative scattering length) causes a condensate to collapse and emit particles (as observed in experiments with $^{85}$Rb in 2001). The scalaron collapse is analogous: when beyond a critical number, the self-gravity (an effective attraction) causes a collapse and emission of scalar radiation (analogous to particle bursts in bosenova)​file-3zh15rq3mb1bnnjszwe2yx​file-3zh15rq3mb1bnnjszwe2yx. In both cases, the system finds a lower-energy state by ejecting mass/energy. Understanding the collapse in scalaron terms could benefit from studies of *critical phenomena in gravitational collapse* (Choptuik scaling, etc.), and from BEC collapse dynamics in a trapping potential. Notably, the presence of the $\phi$–curvature coupling might introduce something akin to a self-interaction term that can stabilize slightly (if repulsive) or further destabilize (if effectively attractive). This parallels how tuning the scattering length in a BEC controls stability. The **critical soliton mass ~0.6 $M\_{\rm Pl}^2/m$** is like the analog of the Chandrasekhar limit or maximum number for a stable bosonic condensate​file-3zh15rq3mb1bnnjszwe2yx​file-3zh15rq3mb1bnnjszwe2yx. Including $\phi^2R$ coupling would adjust this number (for example, an attractive $\phi^4$ self-interaction term can either increase or decrease the maximum mass depending on its sign – similar to how a positive scattering length allows a larger stable BEC number compared to a negative one).
* **Entropy Flow and Thermalization:** In the scalaron’s evolution, we see an **entropy flow from the initially coherent field into gravitational radiation or small-scale fluctuations**, much like energy dissipating as heat. One could say the scalaron field “thermalizes” with the gravitational environment – though not to a high temperature, rather it reaches a **virialized state** where the macroscopic coherence is lost. The concept of a **gravitational temperature** could be invoked: e.g., one can associate an effective temperature $T\_{\rm eff}$ with the random granular motions in region B. This $T\_{\rm eff}$ is not fundamental but emerges from the dispersion in velocities (like assigning a temperature to an N-body system via kinetic energy). If so, $F\_c$ would be 1 at $T\_{\rm eff}=0$ and drop as $T\_{\rm eff}$ increases. This parallels the usual Bose gas where condensate fraction decreases with temperature​[en.wikipedia.org](https://en.wikipedia.org/wiki/Bose%E2%80%93Einstein_condensate#:~:text=fraction%20Image%3A%20,At%20temperatures%20near). The difference is that here $T\_{\rm eff}$ is set by gravitational dynamics rather than an external heat bath. Nonetheless, one could formalize this by saying: at “halo virial temperature” (which can be tens of K if one foolishly equates DM velocity dispersion to a temperature), the condensate fraction is $F\_c$. There might even be a *critical virial temperature* above which no solitonic core can form (if $F\_c$ would go to zero). This is speculative but bridges the gap between cosmological structure and thermodynamic language.

In conclusion, the condensed matter analogies reinforce our understanding of the scalaron phases and transitions. They suggest viewing the **adaptive scalaron as a cosmic superfluid that can undergo phase transitions**. By formulating our model with these analogies in mind, we ensure that it captures not just the gravitational dynamics, but also the *statistical* behavior (coherence vs. decoherence) that is crucial for matching simulations and, ultimately, observations (e.g. distinguishing wave interference effects from classical behavior in halos).

**Conclusion and Outlook**

In this research memo, we developed a coherent analytic formulation for the adaptive scalaron field, embedding the key environmental dependencies directly into the field’s action via curvature and matter couplings. We proposed minimal coupling structures – from simple $\phi^2 R$ terms to chameleon-like $\phi T$ interactions – that endow the scalaron with a density-dependent effective mass, **screening it in high-density regions and freeing it in low-density expanses**. Using these formulations, we explained how the scalaron naturally transitions between a quantum-coherent phase (supporting wave-like phenomena such as solitonic cores and interference) and a decoherent classical phase (resembling collisionless dark matter) as a function of environment. We introduced a **phase-space diagram** that delineates these regimes and identified a **coherence fraction $F\_c$ as an order parameter**, drawing analogies to phase transitions in Bose–Einstein condensates. Crucially, our model is consistent with (and indeed illustrates) the **increase of entropy and the arrow of time** in structure formation: as the scalaron’s coherence is lost, entropy is generated​file-4bzwyu5xwcza2f2huwkyos, aligning with the Second Law and Penrose’s gravitational entropy concept​[link.springer.com](https://link.springer.com/article/10.1007/s10714-023-03070-2#:~:text=Penrose%20argues%C2%A0,a%20low%20value%20value%20of).

The candidate actions discussed will be tested and refined in the forthcoming formal write-up (RFT 10.0). There, we will present detailed equations of motion in both Jordan and Einstein frames, perform stability analyses of homogeneous and solitonic solutions under each coupling, and compare quantitatively to simulation data (e.g. matching the density at which coherence breaks by calibrating $\alpha$ or $\xi$). We will also address open issues such as: might there be observational signatures of the scalaron’s adaptive nature (for example, suppression of small-scale power, or unique gravitational wave bursts from collapse events)? Can the scalaron be extended to include *other effects like torsion or additional fields*, and would that yield any qualitatively new behavior (e.g. a richer phase structure)? And how does the presence of ordinary baryonic matter impact the scalaron’s coherence (since baryons add additional “noise” and potential wells)?

In summary, the adaptive scalaron field – governed by a curvature-coupled action – is a compelling unifying framework for dark matter and modified gravity. It not only reproduces the successes of fuzzy dark matter on galactic scales and chameleon gravity in high-density environments, but it also provides natural explanations for **quantum-to-classical transitions and irreversibility in the cosmos**. By solidifying the action formulation and linking it to measurable quantities (coherence length, halo density, etc.), we lay the groundwork for a testable theory. The next steps will involve confronting this theory with astrophysical observations (galaxy rotation curves, cluster dynamics, gravitational lensing, etc.) to see if the adaptive scalaron leaves subtle imprints distinguishable from $\Lambda$CDM or alternative models. The tools and concepts developed here – coupling structures, phase diagrams, and analogies – will be invaluable in that endeavor, and they mark a significant stride toward the full RFT 10.0 theory of an adaptive, phase-changing cosmic scalaron.